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TRAPPED PARTICLES IN A DISTORTED DIPOLE FIELD

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ABSTRACT

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A dipole magnetic field is analytically compressed and extended to represent the presumed effect of the solar wind on the earth's magnetic field on the day and night side of the earth in the earth-sun meridian plane. The first and second adiabatic invariants of particle motion are used to calculate the mirror points of constant energy particles in the compressed and extended fields. The results support and clarify work by previous investigators, showing that shifts in mirror points of particles moving from one field to the other are large at large distances and smaller at lower altitudes. The results are compared to the experimental findings of Explorers 12 and 14 and Injun 1. It is concluded that field distortion may be a major cause of the shift in mirror points at 1000 km observed by Injun 1, but that final resolution of the problem will depend on further determination of the magnetic field on the night side of the earth.

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INTRODUCTION

Work in recent years has indicated that a solar wind distorts the earth's dipole-like field, compressing it on the day side, extending it on the night side, and confining it to a cavity around the earth. This distortion will certainly affect energetic particles trapped in the field and may help explain the complicated morphology revealed by trapped particle measurements. Of particular interest is whether the distortion of the field can explain results of Injun 1 [O'Brien, 1963] which detected particles mirroring at 1000 km up to approximately 75° geomagnetic latitude on the day side of the earth during magnetically quiet periods, but only up to 69° on the night side of the earth. O'Brien also discussed possible explanations of this effect, including the effect of field distortions.

It is well known that trapped particles spiral along lines of force moving from hemisphere to hemisphere while also drifting in longitude due to the gradient and curvature of the field. The most convenient way of studying these particles is to use the theory of adiabatic invariants of particle motion [Northrop and Teller, 1960]. This theory states that the first adiabatic invariant or magnetic moment μ, defined as $W_{\perp}/B$ ($W_{\perp}$ is the energy due to motion perpendicular to the field and B the value of the field), is constant as long as the variation of B is small during a period of gyration of the particle about the field line. The second adiabatic invariant or longitudinal invariant, J, is defined by $\int_M^{M'} p_{\parallel}\, ds$ where $p_{\parallel}$ is the momentum parallel to the field line, ds is a length element along the field line, and M and M' are the mirror points in either hemisphere. As long as the variation of B over a bounce period is not appreciable, J is constant. As is generally done, these invariants will be assumed constant in the present work, in which case adiabatic theory supplies the interesting features of the particle motion. Also electric fields associated with rotation of the magnetic field will not be considered which means that the energy of each particle will be constant.

The magnetic moment may be written $\mu = W/B_{mir}$ so that the requirement that energy be constant along with the invariance of μ means the particle always mirrors in a field of the same strength. As long as the energy does not change, J may be divided by mv which defines a new quantity I which is constant. This new constant I may be put in the form

$$I = \int_M^{M'} [1 - (B/B_{mir})]^{1/2}\, ds$$

where the integral is along a field line between mirror points of the particles, B represents the field strength along a field line, and B_{mir} the mirroring field.

The above may be summarized in relation to a model field, such as illustrated for the earth-sun meridian plane in figure 1, if one considers a particle with any mirror point on one side of the earth.

When the inhomogeneous magnetic field has caused the particle to drift to the opposite side of the earth, the particle will have a mirror point on the same constant B contour as a result of conservation of μ. The exact position of the mirror point on the contour will be determined by the requirement that the mirror point limits in the I integral be such that I has the same value as on the opposite side of the earth. Once a new mirror point has been located, the line of force through the mirror point may be used to determine the equatorial crossing of the particle.

Various workers [Vestine, 1960; Reid and Rees, 1961; Malville, 1960; and Hones, 1963] have indicated that the effect of compression of the field on the day side relative to that on the night side of the earth will tend to move mirror points of trapped particles toward a lower latitude on the night side. There has, however, been some disagreement as to the magnitude and importance of this effect.

Reid and Rees [1961] used the first adiabatic invariant and realized that particles near the equatorial plane would move in toward the earth on the night side. They further suggested that a correspondingly large effect would result at low altitudes and explain the observations of hydrogen emission in the aurora. Malville used the second adiabatic invariant and a field perturbed by an image dipole and concluded that the latitude shift at low altitudes was not more than 2°.

Hones recently investigated particle motions in a dipole field perturbed by a stronger dipole field at a large distance from the earth's field. He emphasizes the effects of rotation of the magnetic field as well as gradient drifts and obtains interesting motions for particles of various energies. Hones concludes that a latitude shift would be about 3° with his model field although he states, as did Malville, that the effect could be increased by extending lines of force on the night side of the earth.

ANALYSIS AND RESULTS

The present work uses a third type model of a perturbed field to further investigate the effect on trapped particles and, in particular, to examine quantitatively the effect of extension of field lines on the night side of the earth. For the sake of simplicity, the distorted earth's dipole field has been represented analytically by a dipole field to which is added a uniform field parallel to the dipole axes. If this uniform field is southward, the field becomes elongated as it might on the night side of the earth. If the field is northward, the field is compressed as it might be by a solar wind on the day side of the earth. These fields are shown in figure 1. If some smooth transition from the night side field to the day side is assumed, particles will drift around the earth as a result of the inhomogeneous magnetic field. Interesting effects can be analyzed by looking at the particle only when it is in the earth-sun meridian plane.

It should be emphasized that the technique of adding a uniform field to the basic dipole field is essentially a mathematical artifact designed to represent in a simple manner the asymmetric character of the magnetic field in the magnetosphere. It is not necessary, and should not be inferred, that these uniform fields represent the interplanetary field, although discussions along these lines have been given recently by Dungey [1963]. Since the geomagnetic fields produced in this manner are only claimed to be plausible, and are only used in the earth-sun meridian plane, no questions of continuity between them need arise.

Experimental results from Explorer 14 indicate a cutoff in trapped particles at roughly 9 or 10 earth radii in the equatorial plane on the sunward side of the earth. Injun 1 results indicate a cutoff latitude for particles mirroring at 1000 km of about 75° latitude on the sunward side of the earth. In line with these observations a uniform field of 30γ or 1/1000 of the ground value of the dipole was chosen to distort the dipole. This 30γ distorting field results in a compressed line of force from 75° latitude at the earth surface cutting the equatorial plane at 9.2 R_E (as compared to an undistorted dipole line from 75° which cuts the equatorial plane at 15 R_E). Since a boundary has been observed at this approximate position by both magnetic [Cahill and Amazeen, 1963] and particle [Freeman, 1963] measurements, this line will be taken as the outside line of force in this model.

Initially, a 30γ field in the southward direction was chosen to elongate the dipole, but this field may be varied by scaling the calculations. When this southward uniform field is added, the resulting distorted field is capable of trapping particles only in a region bounded by a line which goes through a neutral point in the equatorial plane where the dipole and uniform fields exactly cancel. Any particles outside this limiting line will not be trapped because only one end of the line of force is connected to the earth. With a 30γ distorting field, this limiting line cuts the earth at 67.2° latitude and crosses the equatorial plane at 10 R_E. This limiting latitude prohibits trapping as high as 69°, as observed by Injun 1, but the distorting field may be weakened by scaling so that the limiting latitude moves north.

To apply the above theory it is necessary to calculate I for various mirror points in both distorted fields. This has been done using a modified version of the B-L coordinate computer program developed by E. G. Stassinopoulos at the Goddard Space Flight Center. The results are presented in figure 2. Here the lines are composed of mirror points at a constant latitude with various radial distances as marked on this curve. The B and I corresponding to these different mirror points are given by the ordinate and abscissa, respectively. Solid lines correspond to the compressed field while broken lines represent mirror points in an extended field. Since B and I remain constant for a particle drifting from one field to the other, points of intersection give the mirror point coordinates for a particle in either field.

In the compressed field, I increases with increasing radius r at a given latitude as long as B_{mir} is large compared to B along a large portion of the path of integration. For larger r, however, B_{mir} decreases so that B/B_{mir} along the path is near 1; thus the integrand and, hence, the integral are small.

In the extended field the curves in figure 2 become more horizontal with increasing r at a constant latitude as the limiting line of the cavity is approached. This is caused by lines of force near the cavity limit carrying the particle farther out, causing a longer path of integration and a smaller B/B_{mir}, and thus a larger I.

Changes in the mirror point as the particle drifts from a compressed to an extended field can be determined from figure 2 and are more explicitly illustrated in figure 3. Here the outer field lines for the compressed and extended fields are illustrated. The darkened circles represent mirror points in the extended field, and the adjoining open circles represent the corresponding mirror point positions in the compressed field. Large changes in mirror points occur at large distances as mirror points move closer to the earth to remain in constant field strengths, as pointed out by Reid and Rees [1961]. At lower altitudes the shift is reduced and is more in line with the findings of Malville [1960].

If a particle is to remain trapped for a complete circuit around the earth, it must have mirror points in both extended and compressed fields. Since trapped particles are not found outside the boundary on the sunward side of the earth, there are no particles mirroring in a field of less than about 60γ. On the night side of the earth this means there are no particles mirroring outside about 7 R_E. All particles observed on the equator outside 7 R_E will come from lower altitudes and have smaller pitch angles.

To investigate the distribution of particles crossing the equatorial plane, it is necessary to follow field lines through the mirror points down to their equatorial crossing. Figure 4 shows compressed fields (dotted lines) and extended fields (solid lines) and additional pairs of mirror points. Two effects will influence the equatorial distribution of trapped particles. First, the shift in the mirror points in the night side field will tend to move particles to lower latitudes corresponding to equatorial crossings nearer the earth. Second, the extension of the field lines will carry particles farther from the earth. The dark line in figure 4 represents mirror points on the day side which are shifted earthward on the night side but where the extended field carries the particles out so that they cross the equatorial plane at the same distance as on the day side. For mirror points in the compressed field above this line, the extended field effect dominates and the particles are carried farther out on the night side than on the day side. For mirror points below this line, the equatorial shift dominates and the particles cross the equatorial plane nearer the earth on the night side than the day side.

Hones [1963] concludes that a large percentage of particles injected at the equator with an isotropic distribution of pitch angles will mirror near the equatorial plane. This fact and the much larger area below the heavy line in figure 4 indicate that the large majority of particles will cross the equatorial plane nearer the earth on the night side than on the day side. This is in agreement with results of Explorer 14 [Frank, Van Allen, and Macagno, 1963] which indicate a decrease of two orders of magnitude on quiet days at a distance of 7 to 8 R_E while the daytime intensities are large out to a cutoff at 9 or 10 R_E on the day side.

As was pointed out above, shifts in mirror points are large at large radial distances and smaller at lower altitudes. These shifts at auroral zone latitudes are of particular interest since they have been measured by Injun 1, and they may be related to other geophysical phenomena.

To study these high latitude shifts in greater detail I values were computed for latitudes near the auroral zone. The results are presented in figure 5 which is the high latitude extension of figure 2. Again the two pairs of coordinates associated with any point in the plane (any given B and I) are those of the mirror points in the respective fields. The solid line represents altitudes of 1000 km corresponding to the Injun measurements. The latitude shifts can be determined from figure 5 and are represented by the top curve in figure 6. The maximum shift in latitude can be seen to be about 5°. The southward perturbing field used in the above calculations corresponds to a limiting line of force cutting the earth at 67.2° latitude and restricting mirror points at 1000 km to even lower latitudes. To allow comparison of the theory with the Injun results it is of interest to decrease the strength of the perturbing night side field. This has the effect of (1) moving the limiting line of force to higher latitudes, which allows mirror points in the region found by Injun 2; (2) increasing the radial distance of the neutral point and increasing the extension of the outermost field lines; and (3) making the field inside the outermost field lines more like a dipole.

This weakening of the perturbing field can be accomplished by scaling the calculations made for the above field. A new value for a unit of earth radius (R') may be chosen as some fraction f (less than unity to decrease the perturbing field) of the old unit of earth radius R, that is, $R' = fR$. Any distance in new units r' will be $r' = rR/R' = r/f$. In particular, the distance of the neutral point $r_n' = r_n/f$ will be greater if $f < 1$, corresponding to a decreased perturbing field.

The integral for I is a function only of a ratio B/B_{mir} and the length which enters through ds. The ratio will remain constant when a new unit of length is chosen, and length we have seen is inversely proportionate to f so we also have a new I' given by $I' = I/f$.

When a new unit of earth radius is chosen, B at the new surface of the earth is different and a renormalization of the field is required. Magnetic fields vary as $1/r^3$, so B near the earth's surface is B/f^3. If this is again to be made equal to the original value at the earth's surface, it must be multiplied by f^3 so the new B's become $B' = f^3B$.

This scaling was carried out so that the perturbing field was successively reduced to 18γ and 7.5γ. These fields correspond to limiting latitudes at the earth of 69.1° and 72.1° latitude, and 11.9 and 15.9 earth radii distances of the neutral points, respectively. The latitude shifts at 1000 km obtained with these weaker fields are plotted in figure 6. Reducing the perturbing field decreases the shift in latitude at any fixed latitude, but extending the high latitude field lines makes the shift in high latitudes larger than 5°. It should also be pointed out that the compression of the sun side magnetic field could be increased somewhat and still agree with experimental results. This would further enhance the equatorial shift of mirror points on the night side.

CONCLUSIONS

It appears that the effect of a distorted magnetic field may produce a shift in the latitude of particles mirroring at 1000 km large enough to agree with the experimental results of Injun 1. This conclusion agrees, at least in part, with that of other investigators [Malville, 1960; Hones, 1963] whose model fields predicted only small shifts, but who pointed out that extending the field lines on the night side might produce the observed effect. Final conclusions on this question will have to await the experimental measurements of Explorer 14 and later satellites that will determine which model field is most nearly correct.

Although it appears that distortion effects may be of adequate magnitude to explain the observed results, there are other effects which may be important. An electric field associated with a rotating magnetosphere, such as discussed by Hones [1963], will tend to reduce the energy of particles moving to lower L shells. This effect will be opposite to that produced by the elongation of field lines. Other effects such as the electric fields driving the DS current systems and the magnetic field distortion of a ring current may also be important.

ACKNOWLEDGMENT

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FIGURE LEGENDS

Figure 1. - Magnetic field lines (solid) and contours of constant B (dashed) obtained by superposing a southward directed dipole magnetic field having an intensity of 30,000γ at the geomagnetic equator and a uniform field of 30γ parallel to the dipole axis. The combined field of the dipole and a northward uniform field on the left represent conditions on the day side of the earth. The combined field of the dipole and a southward uniform field on the right represent conditions on the night side. A neutral point N occurs at 10 R_E on the night side.

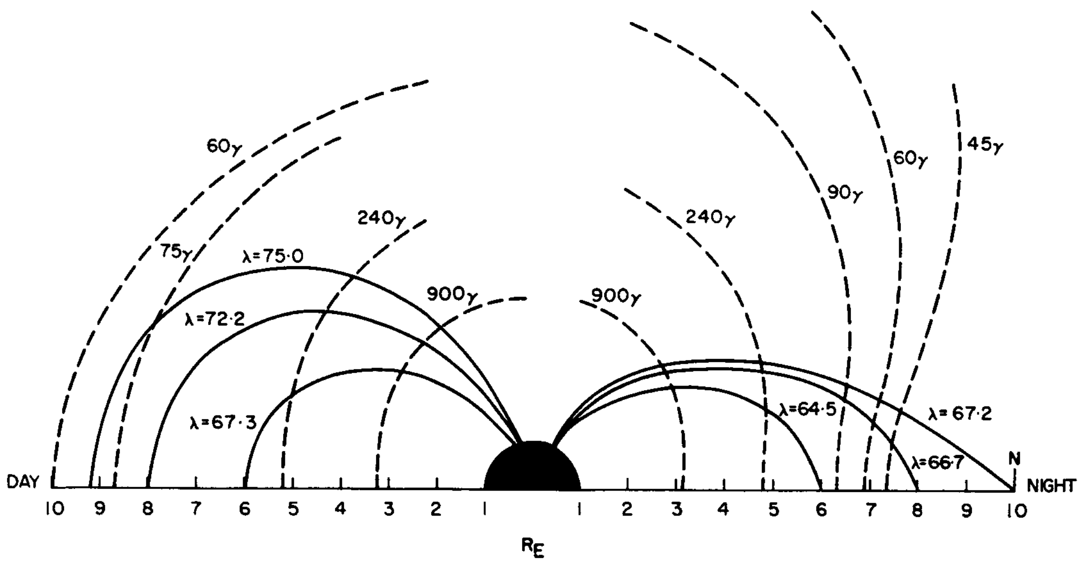
Figure 2. - Curves represent mirror point positions at constant latitudes at distances in earth radii as marked on the curve. Values B and I associated with the mirror points are given by the ordinate and abscissa, respectively. Dashed lines correspond to the sun side field, and solid lines to the night side field. The two pairs of coordinates associated with points of intersection give mirror point positions of a particle which has drifted from one field to the other.

Figure 3.- The outer lines of force for the day side (dashed) and the night side (solid) for the case of 30γ perturbing fields in both northern and southern directions. Open circles represent mirror point positions on the day side of the earth while attached dark circles represent their corresponding position on the night side. Weakening the night side field to improve the agreement with experimental data would move the neutral point N to a greater distance and expand the night side cavity but have little effect on the behavior of the shifts illustrated.

Figure 4.- Dashed lines represent day side field lines and solid lines, night side field lines. Circles again represent the mirror point of a particle in both fields. The heavy line represents mirror point locations on the day side which shift to positions on the night side such that the equatorial crossing of the particles are equivalent on either side of the earth.

Figure 5.- This represents the high latitude extension of figure 2 for 30γ fields in either direction. The horizontal line represents radii at 1000 km. When the night side field is weakened, these values may be scaled and the points inside the earth above become of interest.

Figure 6.- The curves represent shifts in latitude for mirror points of particles moving from the day side of the earth to the night side. The results apply to mirror points at 1000 km and are for several different strengths of the southward night side perturbing fields. The curves are labeled with the strengths of the perturbing fields, the radial distances of the neutral point R_N, and the latitude at which the limiting line of force from the neutral point cuts the earth λ_L.



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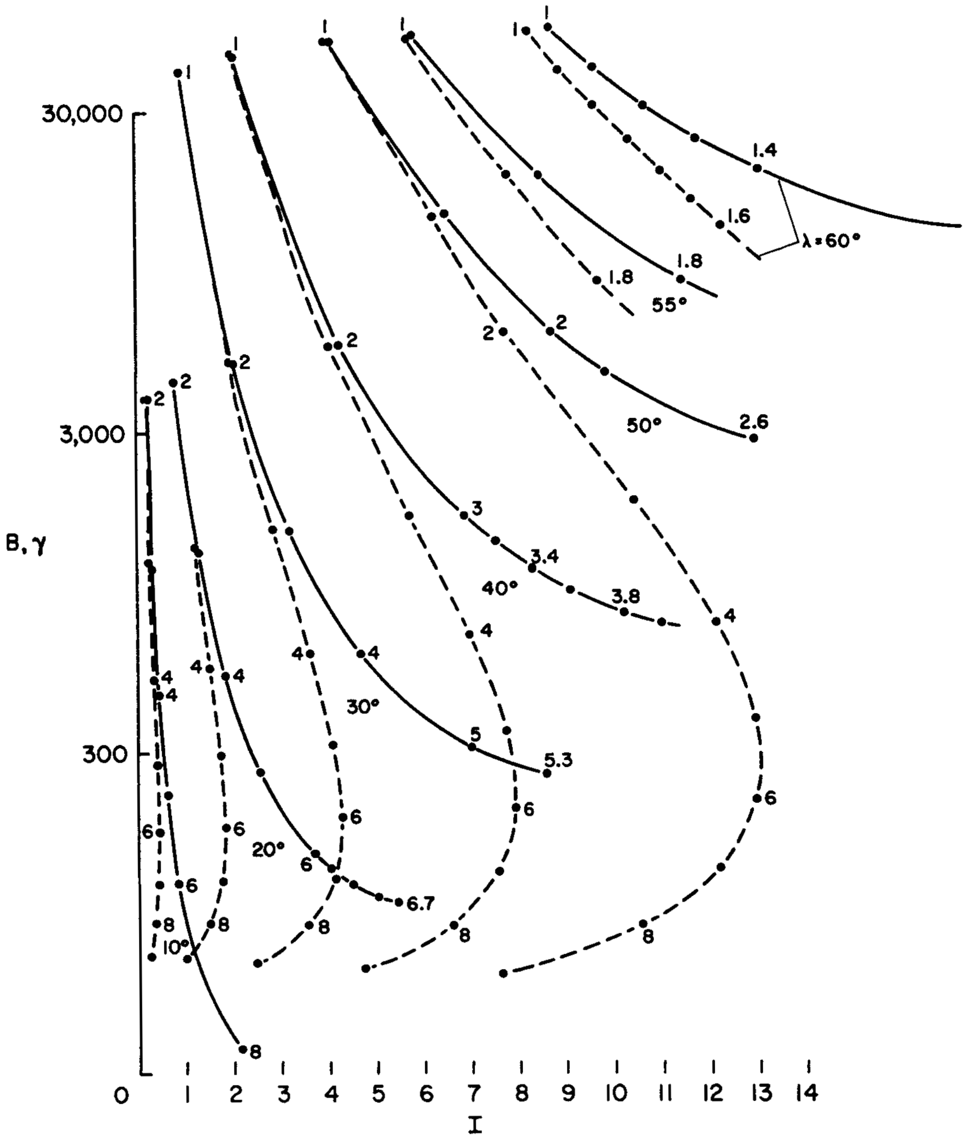

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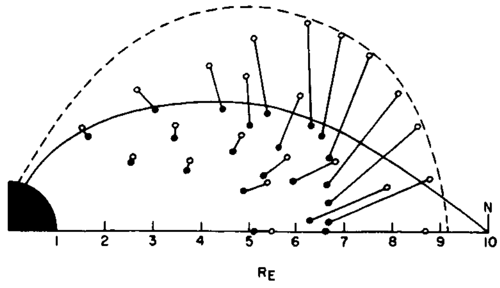

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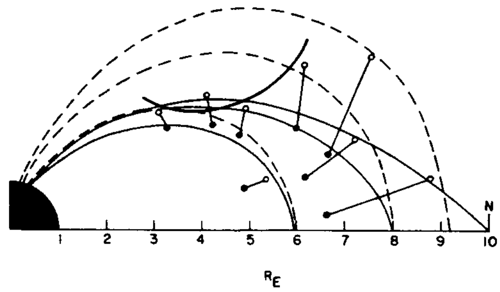

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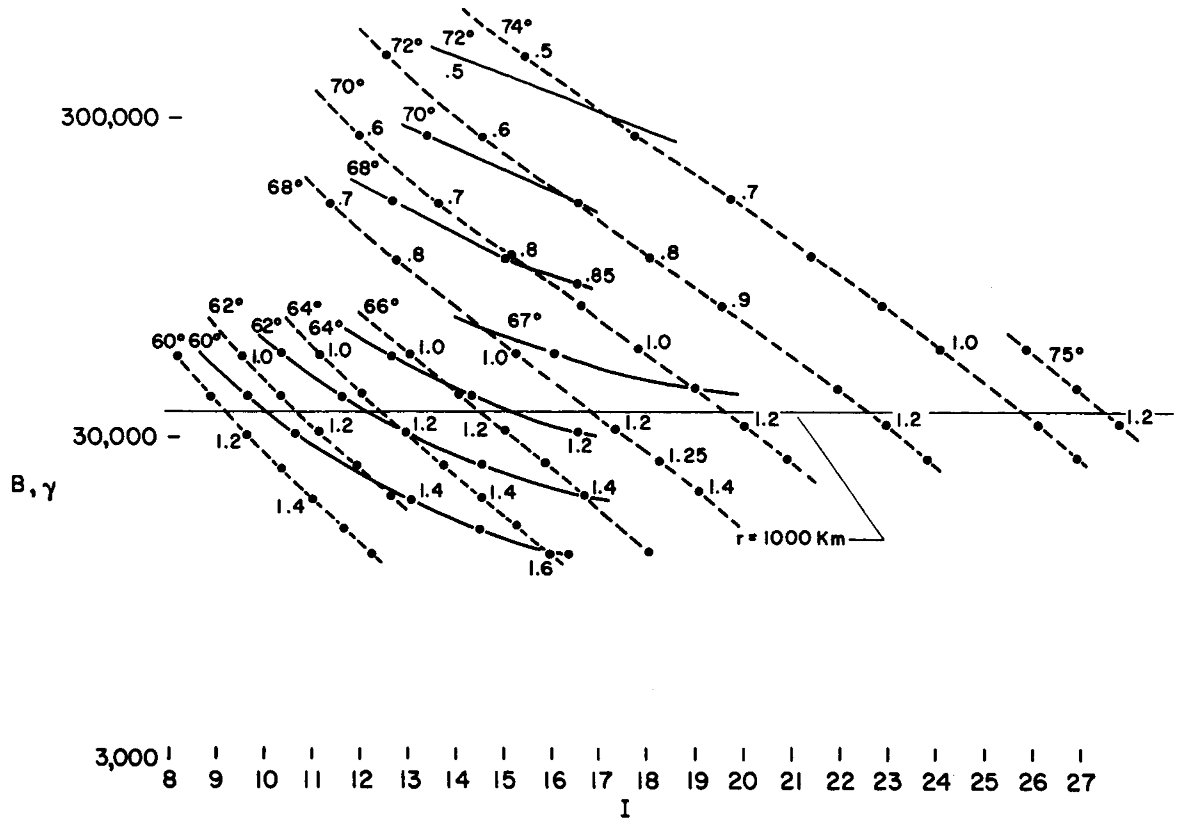

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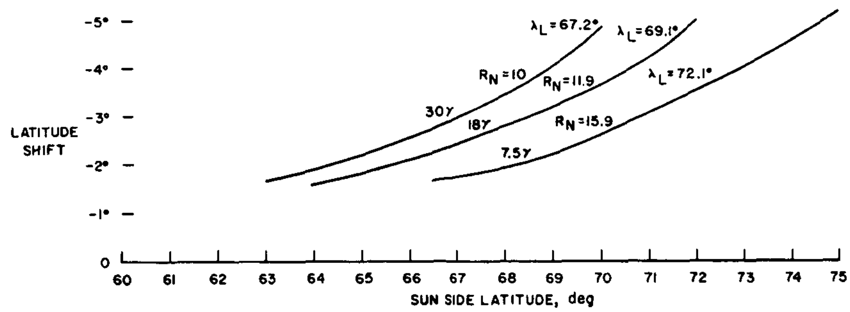

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